fundamental panel mode will be that of an aerodynamic spring, while for any antisymmetric panel mode, such as the second longitudinal mode, only aerodynamic mass effects will exist. The difference in effects is particularly significant in panel flutter where frequency differences are extremely important.

The equation of motion of the plate is

$$D \frac{\partial^4 W}{\partial x^4} + 2D \frac{\partial^4 W}{\partial x^2 \partial y^2} + D \frac{\partial^4 W}{\partial y^4} + m \frac{\partial^2 W}{\partial t^2} = P \quad (8)$$

We shall study the particular case of a panel clamped on four edges. Thus the panel deflection will be represented by a series of the form

$$W = \sum_r \sum_s f_{rs} \psi_r(x) \psi_s(y) \quad (9)$$

where

$$\psi_r = \cos \frac{(r-1)\pi x}{l} - \cos \frac{(r+1)\pi x}{l}$$

These primitive panel modes are not orthogonal; however, for the present purposes we may reasonably neglect the structural coupling and also the coupling between modes due to the cavity. Thus we shall treat each panel mode individually. Eqs. (7), (8), and (9) have been solved by the Galerkin method.

A comparison of the present theoretical results with previously obtained experimental results is presented in Figs. 2 and 3. In obtaining the theoretical curves the experimental values of the in vacuo frequencies were used. In Fig. 2 a plot of frequency vs cavity depth at standard atmospheric conditions is shown. Since considerably lower air densities are encountered in the wind tunnel, the variation of frequency with cavity density has been determined as well (see Fig. 3).

As may be seen, the agreement between theory and experiment is good except at the smaller cavity depths and densities. In particular, the change in the mode of minimum frequency predicted by theory is confirmed by the experimental data, with reasonable agreement on the cavity depth for which it occurs. One may hypothesize that an inclusion of nonlinear and/or viscous effects would bring the theory into better agreement with the experimental results at the smaller cavity depths and densities.

A theoretical calculation of the flutter dynamic pressure has been made for the tested panel with a cavity depth of three inches. The result indicates a decrease in flutter dynamic pressure of the order of 20 percent at Mach 2 with zero pressure differential across the panel.

In many practical cases only the fundamental mode will be modified by the aerodynamic-spring effect and the virtual-mass effect may be neglected. For this case a useful approximate formula may be written for the frequency of the fundamental mode—viz.,

$$K^2 \cong K^2_{\text{in vacuo}} + C\lambda \frac{l}{d}$$

where

$$K^2 = m\omega^2 l^4/D$$

$$\lambda = \rho a^2 l^3/D$$

and C is a constant determined by the boundary conditions. For a panel clamped on four sides $C = 0.44$, while for a simply-supported panel $C = 0.66$.

Reference

[1] Strutt, J. W. (Lord Rayleigh), *The Theory of Sound,* 1st American ed. (Dover Publications, New York, 1945), Vol. 2, p. 69.

Skin-Friction-Work Recovery by Aerodynamic Heating of Skin Coolants

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Nomenclature

c_p = specific heat
c_f = local skin-friction coefficient
c_h = local heat-transfer index (Stanton number)
g = acceleration due to gravity
J = mechanical equivalent of heat
ρ = local flow density
q = local heat-transfer rate
r = temperature-recovery factor
T = local temperature
V = local free-stream velocity
W = skin-friction work rate

Subscripts

aw = adiabatic wall
l = local
w = wall
t = total

THERMAL protection methods in current use on high-speed aircraft and missiles include heat sinks, ablation of coating materials, and transpiration cooling by fluids. To varying extents, the thermal energy transferred to vehicles using these methods of protection is lost. This note gives the result of an analysis that was made to determine the relationship of recoverable thermal energy to drag work when a skin coolant is used as an auxiliary means of propulsion or to implement the thrust of the main propulsive elements.

The rate of heat transfer per unit area on a flat plate can be written

$$q = c_h c_p \rho V g (T_{aw} - T_w) \quad (1)$$

and the rate of skin-friction work in Btu's by

$$W = 1/2 c_f \rho V^3 / J \quad (2)$$

To determine the ratio of the heat recovered by the coolant to the skin-friction work, Eq. (1) is divided by Eq. (2):

$$q/W = 2 c_h c_p g\, J (T_{aw} - T_w)/(c_f V^2) \quad (3)$$

When the Reynolds analogy between the Stanton number and the skin-friction coefficient ($c_f = 2c_h$), as described by Truitt,[1] is used and the expression for velocity is taken from the energy equation, $V^2 = 2c_p J g (T_t - T_l)$, Eq. (3) reduces to

$$\frac{q}{W} = \frac{1}{2} \frac{(T_{aw} - T_w)}{(T_t - T_l)} \quad (4)$$

or, substituting the temperature-recovery factor, $r = (T_{aw} - T_l)/(T_t - T_l)$,

$$\frac{q}{W} = \frac{1}{2} r - \frac{1}{2} \frac{(T_w - T_l)}{(T_t - T_l)} \quad (5)$$

The ratio of recoverable energy to the flow-friction work is shown in Fig. 1 for surface angles of 0°, 10°, and 20° at a free-stream Mach number of 10. While the lower wall temperature shows better efficiency, the jet propulsive efficiency of the coolant decreases as the jet temperature decreases. However, the coolant gains temperature as it flows within

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[1] Truitt, R. W., *Fundamentals of Aerodynamic Heating* (The Ronald Press Co., New York, 1960), pp. 29–30.

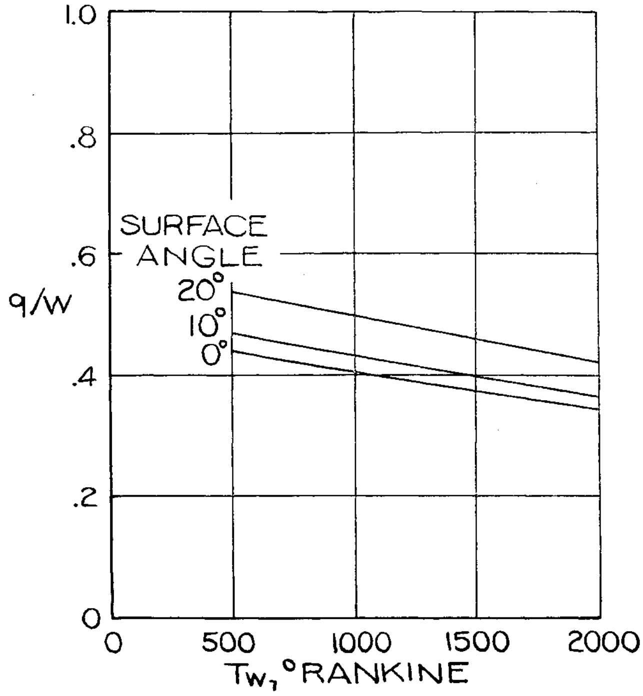

Fig. 1

the skin, causing the skin temperature to vary from a low value at the coolant input to the material-limit temperature at the coolant exit. The recovered energy is, therefore, represented by the average along a coolant tube and is greater than that indicated by the structural material-temperature limit.

A Method of Structural Weight Minimization Suitable for High-Speed Digital Computers

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Let the input for a structural design problem, i.e., lengths, thicknesses, densities, etc., consist of the variables $x_1, x_2, \ldots x_m$ or, in vector form, $x = (x_1, x_2, \ldots x_m)$ and the output be $\sigma = (\sigma_1, \sigma_2, \ldots \sigma_t)$, where each $\sigma_i = \sigma_i(x_1, x_2, \ldots x_m)$ is a function of the x's. The first n_1 of the σ_i may be stresses or deflections, say, at some point of the structure for load condition 1, the next n_1 for condition 2, etc., so that all applied load conditions are represented in the output vector.

The output may have bounds $\sigma^B = (\sigma_1{}^B, \sigma_2{}^B, \ldots \sigma_t{}^B)$, these being in some cases upper and in others lower bounds. If some output σ_i has both, it may appear twice in the output vector. Also define a weight function $w = w(x_1, x_2, \ldots x_m)$, a unit "direction" vector $u = \text{col}(u_1, u_2, \ldots u_m)$, and a gap vector whose ith component is

$$\Delta\sigma_i = \epsilon_1(\sigma_i{}^B - \sigma_i) \qquad (1)$$

where $\epsilon_1 = +1$ if $\sigma_i{}^B$ is an upper bound, and $\epsilon_1 = -1$ if $\sigma_i{}^B$ is a lower bound. The gap vector is so chosen that if a component is negative a limiting design condition has been exceeded.

To start things off, some point x must be selected. It is immaterial at what point this start is made. Then in the neighborhood of this point an approximation is made of the partial derivative matrix

$$R = (r_{ij}) \qquad r_{ij} = \partial\sigma_j/\partial x_i \qquad \begin{matrix} i = 1,2, \ldots, m \\ j = 1,2, \ldots, t \end{matrix} \qquad (2)$$

The calculation of this matrix can be facilitated greatly by use of the "equivalent load" method described later. Also, the gap vector is computed.

It should be noted that, if all the input parameters are altered in the same proportion as the components of some vector u, i.e., if one moves on the input "surface" in the "direction" u, then the change in the vector σ is given by the formula

$$\partial\sigma/\partial u = \text{col}(\partial\sigma_1/\partial u, \partial\sigma_2/\partial u, \ldots \partial\sigma_t/\partial u) = R^T u \qquad (3)$$

The first direction is the normalized gradient of the weight vector

$$u = g^* = g/(g^T)^{1/2}$$

$$g = \text{col}(\partial w/\partial x_1, \partial w/\partial x_2, \ldots, \partial w/\partial x_m) \qquad (4)$$

If none of the gap vector's components are negative, then a reduction in weight is possible, so that the negative of the gradient (steepest descent) is followed until some gap is closed. To estimate the distance in the negative of any direction u that must be moved to close the smallest gap, assume that R will be constant; i.e., that a linear relationship will hold, and use the formula

$$\Delta\lambda = \epsilon_1\epsilon_2 \min_i \left\{ \frac{\Delta\sigma_i}{|\partial\sigma_i/\partial u|} \right\} \qquad (5)$$

where ϵ_2 is the sign of $\partial\sigma_i/\partial u$, and $\Delta\lambda$ is the step length. Since the gaps, in general, will not close linearly with motion along u, a method of successive approximations must be used to close each gap. Actually, more than one gap might close. Suppose r gaps $i_1, i_2, \ldots i_r$ are closed. One then recomputes the R matrix and the gap vector for the new point $x = (x_1 - \Delta\lambda u_1, x_2 - \Delta\lambda u_2, \ldots x_m - \Delta\lambda u_m)$ and also forms a matrix C from the closed-gap columns $i_1, i_2, \ldots i_r$ of the R matrix. Now consider the vector space *SPC* formed from these columns as basis and also the space *SPO* orthogonal to *SPC*. If input is modified in any direction lying in this orthogonal vector space, the gaps will not be altered, at least until linearity is lost. In the present case it is desirable to choose that direction lying in *SPO* which will reduce the weight as fast as possible. This direction turns out to be the projection of the weight gradient upon the vector space *SPO*. It is given by the formula

$$u = v(v^T v)^{1/2} \qquad v = g - C(C^T C)^{-1} C^T g \qquad (6)$$

The next step is to move along the negative of this projected gradient until the next smallest gap or gaps are closed and then repeat the procedure until as many gaps are closed as possible. This gives minimum weight.

The equivalent load method mentioned earlier is based on the following idea. Let the stiffness matrix equation of the structure be

$$Sw = p \qquad (7)$$

where

w = deflection vector
p = load vector
S = stiffness matrix

Then if a small change ΔS is made in S, this equation becomes

$$(S + \Delta S)\bar{w} = p \qquad (8)$$

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